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ABSTRACT

This study of physics problem-solving identifies reasoning mechanisms that enable the problem-solver to achieve the transformation to a physics structure of the problem situation. Elaboration is explored as a mechanism in fulfilling this transformation by providing beginning problem-solvers with elaborations that they failed to infer. A card sorting experiment was employed in which two versions of physics problem descriptions had to be sorted. A comparison is made between proficient and weak students. Findings provide evidence that the reasoning processes in weak students may be qualitatively different from the reasoning processes in proficient students, and that the major problem for weak problem-solvers is not that they do not know problem types but rather that they fail to elaborate on a given situation properly. (Contains 42 references.) (DDR)

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# *The importance of an enhanced problem representation*

ELWIN R. SAVELSBERGH, TON DE JONG &
MONICA G.M. FERGUSON-HESSLER

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# The importance of an enhanced problem representation

*On the role of elaborations in physics problem solving*

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In physics problem solving, recognising a problem's 'physics structure' is an important step towards selecting a proper solution method. If the problem conforms to a prototypical problem closely enough, the problem solver may recognise the corresponding physics structure from superficial characteristics of the problem statement. This bottom up route of activation demands no deep processing. If the problem statement does not trigger the appropriate structure in such a way, some constructive activity will be required to come to a physics structure of the problem situation. The present study identifies reasoning mechanisms that achieve this transformation. We propose that 'elaboration' is an important mechanism in fulfilling this function. Elaborations are reasoning steps that add inferences to the present problem representation. Proficient problem solvers make particular inferences fluently when confronted with the proper information. Thus, they connect previously unconnected pieces of information. Contrastingly, weak problem solvers fail to make proper elaborations automatically, though they have declarative knowledge of the underlying relations. We experimentally tested the effect of providing beginners with elaborations they failed to infer. Our main interest was whether or not a 'given' elaboration would support deep processing, and thus the establishment of a problem's physics representation. We used a card-sorting experiment in which we had two versions of physics problem descriptions to be sorted; an elaborated and a 'minimum' description. The results for proficient and weak students were compared. We found that the elaborations we gave, supported integrative reasoning in proficient students only. Our findings provide us with evidence that reasoning processes in weak students may be qualitatively different from reasoning processes in proficient students, and that the major problem to weak problem solvers is not that they do not know problem types but rather that they fail to elaborate on a given situation properly.

## 1. Introduction

It is a well known finding that novices, when prompted to categorise a set of problem descriptions, do not sort the problems according to solution principles. Instead they sort the cards according to rather superficial similarities between the problem descriptions (Chi, Feltovich, & Glaser, 1981). This finding has been interpreted to imply that the novices have not structured their knowledge in problem schemas. A problem schema is

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supposed to be a LTM structure centred on a solution method, that also comprises generalised problem situation representations. These generic problem representations would then act as activation patterns for the schema, thus providing a link from a concrete situation to the appropriate solution method. The schema construct is not operationally well defined however. As a consequence the interpretation of the empirical results in terms of schemas leaves room for different readings: either the novices don't have their knowledge structured with respect to solution methods at all, or alternatively, they do know some solution methods and generic problem types, but their problem is rather that they cannot map the specific problem at hand to one of the generic problem types they know. This could be caused either by the student's conception of the generic problem type being too specific to be easily matched, or by the student's inability to elaborate on a given problem description to render it to a richer physics representation that would activate the schema. It is our objective to discriminate between the above mechanisms and to judge their relative importance.

There are several studies, both empirical and theoretical, that provide us with evidence that the way the problem is represented mentally, influences the problem solving ability (Elio & Scharf, 1990; Larkin, 1983; Ploetzner, 1995). In a previous study (Savelsbergh, De Jong, & Ferguson-Hessler, 1996) we have identified characteristics of problem representations that differ between problem solvers of different competence levels. In that study we made a comparison between different kinds of experts, and good and weak novices who had to construct and describe problem situations. Among the major results from that study were the greater coherence and consistence of problem descriptions by experts, compared to those by novices, and of problem descriptions by good novices compared to those by weak novices. Coherence, in this context, means that the relations between elements in the situation description are not just implied, but are made explicit. Consistency refers to the absence of contradictions in the description. A related outcome of the previous study was the difference in the use of elaborations: good novices, in contrast to weak novices, elaborated on an initial situation description they gave. The use of elaborations was even more prominent among experts. An important role for the elaborations in the problem descriptions that we have analysed, was to express the relation between the concrete situation at hand and the abstract underlying physics structure of the problem. In order to clarify in what ways coherence, consistency, and elaboratedness of the problem representation may influence the problem-solving process, we will propose a model for the different elements of the problem-solving process in the following section.

The current study focuses on the differences between good and weak novices[1]. From an education point of view, the differences between good and weak novices are particularly interesting, because they may give an insight in what makes some students less successful than others, and that in turn may contribute to improving education. From an experimental psychology point of view the difference between good and weak novices is interesting because both groups received about the same amount of training, and studied the same information, so that differences between the two groups could be attributed to differences in their ways of processing and structuring the information.

### *1.1 The problem solving process*

In this section we discuss the psychological process of solving a physics problem. Our focus will be on 'true' problems, i.e. problems that are not trivial from the viewpoint of the problem solver. This type of problem cannot be solved on the basis of direct recall of

[1] In different studies several meanings have been given to the word novice. In this study novice will refer to first year physics students who have been exposed to the relevant concepts in the domain, and who have prepared themselves to take the final test for the subject recently.

specific explicit features encountered in previous problem solving experiences, but rather it is necessary to recognise the underlying physics-structure of the situation. Though, even in these cases, the final goal is well-defined in physics problems, problem solving in physics shares some characteristics with problem solving in ill-defined domains such as design (Goel & Pirolli, 1992), that are not prominent in traditional fields of problem solving research like arithmetic, logic reasoning, and puzzles. In early work within the information processing paradigm, these problem types were at the focus of research. In this early research two major processes were distinguished in problem solving: understanding and search. Understanding was considered the less interesting of these two and consequently most attention went to search processes (VanLehn, 1989; cf. Newell & Simon, 1972). For problems like the ones we use, the process of understanding the problem is less trivial, and moreover the distinction between understanding and search will be less clear cut than with well-defined, knowledge lean, problems.

Among the features that distinguish physics problem solving from 'simple' problem solving, the most remarkable may be the amount of 'restructuring' that the problem requires (since there are many ways of restating and enhancing the information from the original problem description in ways that make more or less sense from a physics point of view). As a consequence the problem space becomes an intricate structure where a route that leads to a dead end, and thus forces the problem solver to go back, still may result in a modified representation of the problem (Greeno & Berger, 1987; Goel & Pirolli, 1992). A second -related- feature similar between many ill-defined problems and physics problems is the lack of a parameter that would indicate how close one is to a solution. Such a parameter would be a prerequisite to a fruitful application of weak general problem solving strategies such as hill-climbing[2].

The process of restructuring the problem implies that, between reading the words of the problem description and finding a solution to the problem, the words and propositions from initial problem representation become connected somehow, with the addition of knowledge from LTM, to form a more or less structured mental representation of the problem. A structured representation implies a rich (i.e. redundant) mental encoding of the situation. Such a rich encoding provides more cues to select the proper solution representation, and, moreover, is more robust (i.e. less vulnerable to inconsistencies). We will refer to these added assertions by the name of *elaborations*. Following VanLehn (1989, p.539), we define elaborations as follows: '*An elaboration is an assertion that is added to the state without removing any of the old assertions or decreasing their potential relevance*'. VanLehn further comments that for problem types where elaboration plays an important role, the distinction between understanding and search becomes blurred. The process of elaboration on the initial problem representation is also known by the name of 'deep processing': Craik and Lockhart (1972), for instance, in their classical paper on 'levels of processing', argue that:

[2] The accepted definition of a well-defined problem takes into account whether the initial state, the final state, and the operators are well-defined (Bunge, 1967 p.137; Landa, 1969 (cited in Mettes and Pilot, 1980, p.46); Simon, 1973; VanLehn, 1989). To the psychology of problem solving these parameters might not be the most important ones however. Chess problems are well-defined according to this definition, but still, because of the large number of moves in between the initial and the final state, and because of the large number of operators (moves) that can be applied, knowing the desired final state and all legal moves is of little help. There may be more psychological importance to whether the sub-goals are well-defined; in chess -and in physics- generally they are not.

> *[The] conception of a series or hierarchy of processing stages is often referred to as "depth of processing" where greater "depth" implies a greater degree of semantic or cognitive analysis. After the stimulus has been recognised, it may undergo further processing by enrichment or elaboration. For example, after a word is recognised it may trigger associations, images or stories on the basis of the subjects past experience with the word. Such "elaboration coding" [...] is not restricted to verbal material.* (Craik & Lockhart, 1972, p. 675)

Following Craik and Lockhart we will use the term 'depth' to refer to the amount of processing rather than to an inherent property of the problem representation itself. When we refer to the 'physics relevance' of the problem representation (which is, in the given context of physics, an inherent property of the problem representation itself), we prefer to distinguish between naïve and physics representations (following Larkin, 1983), rather than the more confusing 'surface' versus 'deep' structure as proposed by Chi et al. (1981). That the two dimensions are orthogonal is illustrated by the following: on the one hand, in the case of a kid on a merry-go-round, the observation that the centripetal force is equal to the friction force is just as deep (in the Craik and Lockhart sense) as the observation that the kid will become dizzy. The goal, and the prescribed context of physics theory make one observation a naïve observation and the other a physics one. On the other hand, in a problem where one is asked to compute the centripetal force for a point mass orbiting at a given radius from the origin and with given angular velocity, there is nothing deep about the physics structure of the problem.

If the problem description is textual[3], the initial mental encoding will closely follow the structure of the text. In the course of restructuring the problem representation, the mental representation will take a structure that becomes less dependent on the textual description and that will be more dominated by the structure of the situation being described. This process has been recognised in model of language comprehension too. Kintsch for instance in his construction integration model proposes the following phases in the initial comprehension process:

> *a) forming the concepts and propositions directly corresponding to linguistic input; b) elaborating each of these elements by selecting a small number of its most closely associated neighbours from the general knowledge net; c) inferring certain additional propositions; and d) assigning connection strengths to all pairs of elements that have been created.* (Kintsch, 1988, p.166)

Though the importance of elaborations in problem solving is clearest for complex semantically rich problems like physics problems, elaborations may play an important role in other domains also. Logical reasoning, for instance, is a domain where much emphasis has been on situation-model based reasoning. There is some evidence however, that, also in this domain where the understanding of the problem has been taken for granted traditionally, restructuring the problem and elaborating on the original problem statement may play an important role in solving the problem. Polk and Newell (1995) presented a reanalysis of reasoning protocols obtained with categorical syllogisms that were presented verbally. They propose that the reasoning process in these cases could be equally well be explained in terms of verbal reasoning, i.e. repeated verbal reencoding, instead of manipulation of a situation model. They suggest that mental models do play a role in problems that demand the use of external knowledge and in problems that are presented in a visual format, but that in verbally presented

[3] If the initial problem description is in a pictorial format, things are a bit more complicated for pictures can be recognised as a whole

problems that require no external knowledge, verbal reencoding of the problem may suffice.

We have argued that the mental representation of a problem, that initially consists of a sparse constellation of isolated propositions, evolves into a much richer structure. In many studies it has been proposed that this evolution is controlled by a separate cognitive process, that requires a specific kind of control knowledge (such as metacognition, strategic knowledge etc.). There are many practical and conceptual problems involved in models that employ such a general control mechanism (Baddeley, 1986, Dennett & Kinsbourne, 1992), moreover several studies have shown that content knowledge itself can organise the reasoning process (Anderson, 1983, Rumelhart, McClelland, & the PDP research group, 1986). Since in this study our interest is after the role specific kinds of domain knowledge play in reasoning, we will choose the second approach stressing the associative structures in LTM that give rise to a self-evolving mental model.

Next to reasoning about the problem situation, another aspect of problem solving is reasoning about the solution method. Both the situation and the solution method can be represented in the mind simultaneously. We call these structures the situation representation and the solution representation, respectively. Both the representation of the situation and the representation of the solution method may become active structures that guide further reasoning. The situation representation supports mental simulation, whereas the remembered solution method is a guide to formal problem solving. We assume that there is a close connection between the two in proficient problem solvers who have their knowledge organised in problem schemas that involve both generalised situation models and solution approaches.

The process that we have described here is mainly data-driven initially. In the course of reasoning about the problem there is a shift towards schema-driven reasoning. This shift may either occur gradually, or as a moment of 'insight' that comes in an all or nothing fashion, comparable to the recognition of a Gestalt[4]. In the physics problem solving context, the Gestalt would represent a meaningful type of problem. As soon as a 'schema-structure' is matched sufficiently well, the instantiated schema may direct further reasoning. This transition is a critical event in the problem solving process.[5]

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[4] A notable difference between the types of problems that were used in the traditional Gestalt psychology (like Duncker's (1945) radiation problem, or Maier's (1931) two string problem), and the physics problems that we used, is that in the traditional Gestalt problems, there is a very direct relation between elaborations and problem solving actions: elaborations always related to 'how to use the properties of an object' — like for instance the scissors that can be used as a mass to make a pendulum swing. In physics problem solving in contrast, understanding the situation is a goal in itself and it can be useful to enhance the mental model of the situation even without directly finding a formal solution method.

[5] With the development of expertise, the problem schema structure will become increasingly powerful. As a consequence the schema will be matched more easily and more rapidly, so that less data driven reasoning will be required for common problems. This transition is quite comparable to the knowledge encapsulation process that has been observed in physicians (Boshuizen and Schmidt, 1992)

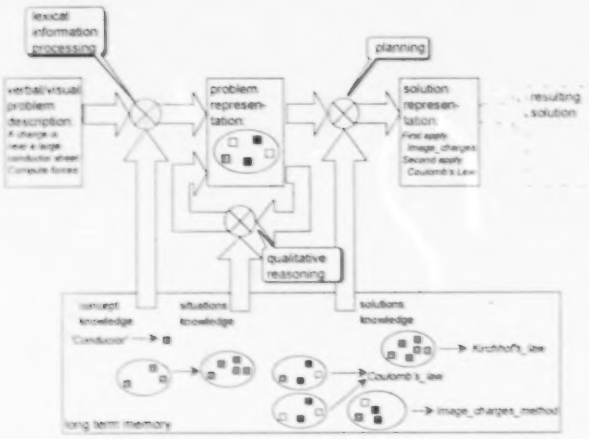
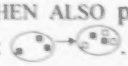
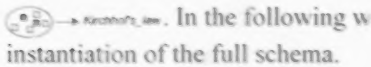



*Figure 1 Model of the states, processes and information flows in the problem-solving process. In the bottom rectangle, the three types of LTM associations are depicted.*

These considerations have led to a structure model of the problem-solving process that is depicted graphically in Figure 1. Claim of the model is not to give a chronological account of the problem solving process but rather to delineate the states, processes, and information flows that play a role.[6] In the model three subprocesses can be distinguished: the processing of lexical information, the qualitative reasoning process that elaborates on the problem representation, and the planning process that results in a solution representation. Beyond these steps, the implementation of the solution would follow. The process leading to the implementation of the solution is beyond the focus of our research, and has been omitted from the model. Each of the subprocesses in Figure 1 corresponds to a particular type of knowledge structure. The first process, lexical information processing, demands knowledge of how words map to concepts. In Figure 1 this type of knowledge is represented by: 'Conductor'→□. The second process: qualitative reasoning, elaborates on the problem representation, by adding matching elaborations to the current model of the problem. These inferences may be represented in LTM in the following format: 'IF property A [AND property B] APPLIES THEN ALSO property C APPLIES'.[7] In Figure 1 this type of knowledge is represented by:  . The third process, finally, makes an appeal to knowledge of solution methods. Again, knowledge of this type comprises a certain pattern of situational properties that is to be matched sufficiently well in order to activate the solution information. In this case however, the corresponding production would be of the format: 'IF property A [AND property B] APPLIES THEN APPLY method C'.[8] In the figure, this type of knowledge is represented by:  . In the following we will focus our attention to what happens prior to the instantiation of the full schema.

[6] In the actual problem solving process the processes outlined in the model will often alternate, rather than being executed in a simple linear fashion (Chi, Glaser, & Rees, 1982; De Jong, 1986; Hayes & Simon, 1974).

[7] Alternatively the mental model of the problem may be thought of as a pattern rather than a verbal statement. The LTM representation of the inferences would then be triggered by a matched activation pattern rather than a formal condition. From that viewpoint the rule-format that is presented here, may be accepted as an approximation to the 'real' structure (cf. Smolensky, 1988).

[8] It is important to recognise that, a primary difference between reasoning about a problem situation, and reasoning about a solution method is that both processes have a different focus. In the former process, the physical system is the subject, whereas in the latter process, the reasoning process operates on solution procedures. Equivalently: in the former process, the question being posed is after a property of the system, whereas in the second process the question being posed is: how to *determine* a property of the system. The two questions are closely related but not equivalent.

## *1.2 Elaborations*

As we have argued in the previous section, elaborations (or equivalently: inferences) play an important role in problem solving. In this study, our specific interest is in the role elaborations play in qualitative reasoning, especially during the initial phases of structuring and restructuring the information from the problem description to form a coherent mental representation of the problem. Reasoning in this initial phase is in part automatic, and reasoning episodes may have a duration as short as a few hundred milliseconds (Shastri & Ajjanagadde, 1993), consequently the depth of such reasoning is quite limited. This type of reasoning is known under such names as on-line inference (Noordman, Vonk, & Kempff, 1992), or reflexive reasoning (Shastri & Ajjanagadde, 1993). Due to its automaticity, the process that leads to the initial interpretation of the situation cannot be captured in think aloud protocols (see also: Ericsson & Simon, 1993; de Groot, 1965). In most cases, at least with novice problem solvers, reflexive reasoning will not suffice to trigger a problem schema, in which cases a more deliberate qualitative reasoning effort will be required. This effort will lead to more complex inferences being made, and deeper conclusions being drawn.

In line with our model that stresses the associative nature of reasoning, we propose that there is no reason why elaborations should be activated in an all or nothing fashion. Rather, we adopt a mechanism like the one described in the ACT* model by Anderson (1983) stating that with a certain node in a knowledge network, a certain activation strength may be associated. According to such a model activation spreads over the knowledge network in LTM, and the activation of a subsequent elaboration would be inversely proportional to the number of elaborations accessible from the initial knowledge element.

In order to gain an insight in the types of inferences that can be made, and in the role they play, we have analysed a number of physics problems. For each problem a hypothetical 'reasoning trace' was constructed, consisting of all the -atomic- reasoning steps that would be necessarily made to be able to plan the solution procedure for the problem. Underlying any conclusion to be drawn we have posited a generalised rule; the 'inference rule'. These inference rules range from very simple, broadly applicable rules, to complex, highly dedicated rules. The conclusion that a body is a conductor, given that it consists of copper, is an example of a simple inference, which may be represented in the following rule:

IF medium(O)=copper APPLIES THEN ALSO medium(O)=conductor APPLIES

This inference can be said to be at the lowest level of complexity, because it is a one-to-one mapping of a statement from the problem description to a new statement. A somewhat more complex inference may be drawn when two objects are concentric, with one of the two having a smaller radius than the other has. The conclusion that the first object is inside the second object may be drawn according to:

IF concentric($O_1$,$O_2$) AND shape.radius($O_1$)<shape.radius($O_2$) APPLIES
THEN ALSO position($O_1$,$O_2$)=in APPLIES[9]

This type of inference may be said to be at a higher level of complexity, because it demands the simultaneous combination of two bits of information to infer something new. Likewise, we could go on with higher levels of complexity. In our analysis we found several examples of statements that integrate five bits of information from the original problem statement in order to reach the next conclusion. Typically, these

[9] 'shape.radius($O_1$)' Denotes the 'radius' aspect of the 'shape' attribute of object $O_1$. One may argue that the two objects both should have a regular shape in order for the above rule to be valid; adapting the rule accordingly would render it an even more complex rule.

conclusions could be reached via intermediate reasoning steps that combined only two or three pieces of information at a time. In some cases, however, no intermediate steps could be defined, and in these case a truly complex inference results. We may suppose that these highly complex inferences would correspond to impasses in the problem solving process. A typical -hypothetical- reasoning trace tends to start with simple inferences, and proceeds with more demanding inferences being made when reasoning progresses. Two hypothetical 'reasoning traces' are presented in appendix A[10].

The inferences that we have discussed tend to combine different pieces of information to generate something new. As became apparent from the foregoing, there are more and less complex inferences. It may be conjectured that complex inferences can be made only on the basis of the simultaneous awareness of all relevant problem features, and that thus because of the limitations of working memory, complex inferences require an integrated representation of the problem situation. On the other hand as a result of making inferences, relations between elements in the problem situation will be established, leading to a more integrated problem representation. Thus inferences may be seen as transformational devices, that integrate a collection of initially isolated propositions into a structured mental model.

Now that we have shown that inferences can be important steps in restructuring the problem representation, the next issue to address is under what conditions these inferences are made. There are several factors that determine when inferences are made, and what inferences are made. Some factors lie in the person, whereas other factors are determined by the input material, such as a problem description. An important factor determined by the problem solving person might be the goal the person has in mind. Some evidence that this factor not only influences conscious reasoning efforts but also the more automatic reflexive reasoning, comes from a study by Noordman et al. (1992), who addressed the influence of the goal a reader has in mind while reading a text. They showed that the reader's goal determines what inferences are made on-line while reading.

Evidence with regard to the effect that properties of the input material can have on the reasoning process, is found in studies by McNamara, Kintsch, Butler Songer, and Kintsch (1996) and Reder, Charney, and Morgan (1986). In both studies, the effect text coherence has on the reasoning process, was investigated. McNamara et al. (1986) compared learning from more and less coherent study texts by high and low prior knowledge subjects. Two types of learning outcome were distinguished: recall of the text base and the quality of the situation model subjects had constructed. They found that the less coherent text impaired recall of the text in both high and low knowledge subjects. In contrast, the acquisition of a situation model was promoted by presenting a less coherent text for high knowledge subjects but not for low knowledge subjects. So, from this study we have also evidence for another factor internal to the problem solver, namely prior knowledge. The findings are interpreted as evidence that high knowledge subjects are able to infer the missing relations themselves, and that the process of inferring helped them to engage in actively constructing a situation model. Reder et al. (1986) manipulated the elaborateness of texts too. They made different versions of a manual to a computer task. They had two types of elaborated texts: one version had 'conceptual elaborations' the other version had procedural elaborations (=syntax elaboration). The version with syntax elaborations led to superior performance on the

[10] This type of 'reasoning' bears strong resemblance to traversing the problem space by executing productions using a breadth-first strategy. There is a difference in that there is no 'operation' that changes the state of the system like there is in most puzzle problems. A second important difference is that in the present type of reasoning the original problem description is not replaced with a modified one, but instead the problem description becomes enhanced with some new information.

task whereas the conceptual elaborations had no effect. Though McNamara et al. (1996) did not distinguish between conceptual and procedural elaborations, from the examples they give we may conclude that their manipulation afflicted the amount of conceptual elaborations in the study text. Thus, the effect of adding conceptual elaborations to a text may have different results dependent on the type of the task and the performance measure that is used.

### *1.3 Research questions*

Now that we have discussed the problem solving process, and the role elaborations may play in this process, we are ready to address our major questions: what makes beginners perform worse than experts when it comes to recognising the type of solution a problem requires, and next: what makes good beginners perform better than weak beginners on this task? We started this paper by describing two possible reasons why novices fail to recognise the proper solution type for a particular problem. Either they don't know any solution types at all, or the core problem to weak problem solvers is not that they have no schemata, or that they do not see solution principles as important, but rather that they fail to translate between the problem at hand and the problem types they know. Our first hypothesis is that, for the kind of novice we are studying (i.e. students who have attended an initial university level course and who have attempted to pass the test), the latter is true.

A powerful method for investigating subjects' schemata is the use of categorisation tasks. In this type of task the subjects have to sort a pile of problems according to the similarities between their solutions. This task cannot be completed successfully using a mechanical trial and error approach. In contrast, it is necessary to recognise the global structure of the solution without actually executing all the steps, which is exactly the kind of reasoning we are interested in. Therefore, in the present experiment, we used the method of card sorting.

If it is true that even weak beginners basically know the types of solutions prevalent in the field, we should expect them to name their problem categories accordingly when they are trying to sort problems according to solution methods. Therefore, our first hypothesis predicts that a typical -weak- beginner sorting would not be systematically different from the experts outcome, but rather the -weak- beginners should be expected to come up with a something like a blurred image of the experts sorting.

This kind of experiment, with similar subjects, has been done by Chi et al. (1981) too. They found major qualitative differences between expert sortings and novice sortings. However, in their sorting experiments they had only few subjects, so that they could not carry out a statistical analysis. Moreover, as pointed out by Taconis (1995), their study has some methodological limitations, both due to the instruction they gave to their subjects and due to the problem set they used.

With respect to the instructional format, it should be noted that Chi et al. (1981) did not give any indication according to what criteria the problems were to be sorted. Differences in instructional format may explain for the mixed findings in other studies too. Gruber and Ziegler (1995) for instance found that chess novices, when categorising chess positions, labelled their clusters qualitatively different from the experts' labels. In the Gruber and Ziegler experiment the instruction gave no clue as to what kind of categorisation was desired. Taconis (1995) had high school students to sort physics problems. He found different results dependent on the instruction he gave.

With respect to the second factor: the composition of the problem set, it is important to notice that Chi et al. used a problem set with 'misleading' cover stories in their

experiment. In their article they reported on two problem sorting tasks. The first task (with 8 expert and 8 novice subjects) lead to the conclusion that the novices gave different names to their clusters. In the second version of the task (where they report on 4 subjects of different expertise levels) they deliberately choose their problems in such a way that the surface properties of the problem suggested a different ordering than the ordering based on solution procedures. In this experiment they found that the novice subject responded to surface properties, the expert responded to 'deep' properties, an advanced intermediate sorted according to solution principles but erred sometimes, and a less advanced intermediate subject made some hybrid sorting distinguishing piles both on the basis of solution principles and surface properties. In physics, however, unlike in algebra word problem solving, the 'cover story' of the problem is not normally independent of the deep structure. Problem statements in traditional university level physics text books in general go little beyond the relevant physics context. The problems used in our experiment were formulated accordingly. In many recent text books (see for instance: Halliday, Resnick, & Walker, 1993; Young & Freedman, 1996) practice problems are embedded in a 'human interest' cover story, which is also the way in which problems were presented in the experiments by Chi et al. (1981).[11] It is conceivable that this type of cover stories conveys a kind of information that is particularly salient to beginners, and that it is this type of information that caused Chi et al.'s subjects to induce the superficial sorting criteria.

If our first hypothesis proves to be true, i.e. if novices appear to know some rudimentary form of a problem schema, their failure to recognise the proper schema in many cases could be caused either by their conception of the generic problem type being to narrow or because they fail to elaborate on the current problem properly. If the elaborations play the role we suppose they do, it may be supposed that, dependent on the subject's level of expertise, some inferences are made automatically upon first sight of a problem. Other inferences are simply too complex and will never be made, and even if they were made (or told) they would not trigger any relevant thought because they are not connected to anything known. In between these two extremes, there must be an intermediate level where making inferences no longer is unproblematic. It may be supposed that presenting someone with an elaboration of this level of complexity may provide a scaffold, that enables the problem solver to solve a more complex problem. If the level of the given elaboration is too high, it will not connect to anything, and thus it will not do any good. If the level of the given elaboration is too low however, the given information may even hamper the active reasoning process, and thus the formation of a situation model (McNamara, 1996).[12] Whether an elaboration is too easy, too difficult, or has the proper level, is dependent on the proficiency of the problem solver. If both good and weak beginners do engage in this kind of elaborative reasoning, we should thus expect that when elaborations are given at a very simple level, weak beginners profit more than good beginners do, which is our second hypothesis. In order to test this hypothesis with made another version of our problem sorting task with an extra elaboration given with

[11] This practice may be aimed at transfer sometimes, but in other cases the mapping between the cover story and the underlying physics template is the more trivial part of solving the problem, suggesting that the sole purpose of the cover story is to give colour to the problem description. As an example consider the following problem where most of the cover story can be neglected: ***What Shakespeare Didn't Tell Us***. *Romeo is tossing pebbles at Juliet's window to wake her. Unfortunately she is a sound sleeper. He finally throws too large a pebble too fast. Just before crashing through the glass, the pebble is moving horizontally, having travelled horizontally a distance* x *and vertically a distance* y *as a projectile. Find the magnitude and direction of the pebble's velocity as it leaves Romeo's hand.* (Young & Freedman, 1996)

[12] There are profound similarities between our model and Vygotsky's theory on the zone of proximal development (Vygotsky, 1978). In our approach there is a range from the simplest elaboration that starts to be helpful to the most complex that is still helpful. This range may vary between persons. It may be supposed that subjects who are able to take profit from a wide range of elaborations will learn in the domain more easily than subjects who are able to take profit from a small range of elaborations only.

each problem. We adjusted the elaborations we gave to be as simple as possible, that is to say: they were very close to the original problem statement.

## 2. Experimental procedure

### *2.1 Subjects*

Subjects were first year physics students who had completed an initial course on electrodynamics in vacuum. Based on power considerations, it was estimated that we needed about 80 subjects in order to establish the interaction between the effect of elaboration and the student level. Subjects were recruited from two different universities (hereafter 'University A' and 'University B'.), because the population within each university was too small to provide us with the subjects we needed.[11] First-year students selected from the faculty's phone directory were approached by telephone until the desired number of 80 participants was reached. Subjects were paid *f*20,- for their participation.

Students were classified as good or weak students on the basis of past test results. Both high school final examination grades and the scores obtained thus far on several university physics tests were available. In earlier research we have already studied the coherence among several grades (Savelsbergh et al., 1996). After comparison with the current set of data, we dropped high school biology from the scale because correlations to the other grades were rather low.

Due to the use of subjects from two universities, the university physics test results were not directly comparable between members of the different subgroups. The problem was resolved by first standardising all scores into z-scores; then the means and variances on the high school final examination scores were determined for each subgroup. It appeared that the variance was about equal for both subgroups, but that the mean of the high school final examination scores for students of university B was significantly higher. With the help of this information, the university test scores for both universities could be matched. Due to the construction procedure of the scales from subscales, we are left with reliability coefficients for the subscales and for the combination of scales. As a reliability coefficient Cronbach $\alpha$ was used. The following values were obtained: high school final examination: $\alpha = .86$; test scores university A: $\alpha = .88$; test scores university B: $\alpha = .93$; the reliability of the resulting combined scale: $\alpha = .88$. The latter value is considered to be quite acceptable. On the basis of the resulting scale, a median split was carried out, resulting in equal sized high and low performance groups that were used for further analysis

The present experiment could be carried out only when in the course on electricity and magnetism all the relevant material had been covered. In order to make sure that the students had really spent some time on the topic so that they would at least understand all the words in the problems descriptions, the experiment could not be carried out before the final test had been taken. For University A this implied that we could start

[11] There are some differences between the universities, and between their respective populations, that should be noted here. At University A, the course on electrodynamics is placed in the entrance semester, at University B, electrodynamics is taught in the second semester. This difference may influence the attitude the students have towards physics learning at the time they studied the subject. A second difference is that at University A, first, both magnetism and electricity are treated for in-vacuum systems. This course is followed by a second course that includes the presence of dielectric materials. At University B, in contrast, electricity, with and without dielectric is covered in a first course and in the sequel to this course, magnetism gets full coverage. A third difference between the two institutes is that University A is a polytechnics institute, whereas University B is a general university.

experimenting after the first semester had been completed. Due to practical reasons there was a lapse of three months between the first opportunity to take the final test and the start of the experiment. In fact, during this period, some of the subjects had taken part in the second opportunity to do the final test already. These circumstances may have caused some extra 'noise' in the data. At University B the coverage of the relevant topics extended over two courses that spread over the entire second semester. At this university the experiment was started immediately after the final test of the second course.

## *2.2 Material*

As a subject matter in this experiment we chose the field of electricity and magnetism. The particularities of this domain compared to other domains are discussed in Savelsbergh et al. (1996).

### 2.2.1 The problem cards

In order to test our second hypothesis, regarding the effect of elaborations, two different sets of problems were required, to let each student work on an elaborated and a non-elaborated version of the task. We constructed two sets of problems from the domain of electricity and magnetism. One set comprised 20 problems from the field of electricity, the other set consisted of 20 problems on magnetism. Our intention was to design both sets in such a way that an expert would distinguish four different clusters of problems in the set. We tried to make the problems within a cluster sufficiently varied, both in their wording and in their physics content, the constraint we used was that we did not want to include catch problems of types the students did not regularly encounter in their practice problems. The design procedure started from a larger set of problems that was presented to several experts[14]. Problems that were not classified in a more or less consistent way were removed from the set, as well as problems that according to the experts were too hard for undergraduates. The composition of the remaining sets is summarised in Table 1. For the resulting two sets of 20 problems each, it was determined, on the basis of the judgement of four independent experts, which problems were never associated with each other, and which problems were almost always (i.e., by at least 3 of the experts) associated, the results are summarised in appendix B. These results were the standard against which the students' sortings were judged.

*Table 1 The distribution of the problems over topics according to the experimenters.*

| *Set 1*: Electricity | Number of problems | *Set 2*: Magnetism | Number of problems |
|---|---|---|---|
| Gauss' law | 6 | Ampere's law | 6 |
| Image charges | 5 | Dipole approximation | 5 |
| Dipole approximation | 5 | Induction/flux | 5 |
| Coulomb's law/superposition | 4 | Biot-Savart's law | 4 |

For each of the problems an elaboration was constructed that had a low level of complexity. So of both sets there were two versions; one with and one without elaboration. All students were to sort one of the sets with elaborations and the other set without elaborations. Examples of the problem cards are given in Table 2.

[14] An expert was defined as someone who had been involved in teaching the subject of electrodynamics at the undergraduate level recently.

*Table 2 Two examples of the problem cards used in the experiment. The non-elaborated version gave only the normal printed text, the elaborated version also included the text in italics.*

| | |
|---|---|
| E17 (intended cluster: Gauss' law) Compute the field of a planar charge distribution that extends to infinity<br><br>*The field of an infinitely large planar charge distribution has a field component perpendicular to the plane only* | M2 (intended cluster: induction/flux) A coil with radius r, length l and N turns, is being rotated at angular velocity ω in a homogeneous magnetic field. The axis of rotation is perpendicular to the field. Compute the voltage induced in the coil.<br>*As a consequence of the rotation the magnetic flux through the coil changes* |

In order to average out effects of order; the following design was chosen (Table 3).

*Table 3 The experimental set-up*

| | first with elaboration | first without elaboration |
|---|---|---|
| proficient student | | |
| first electricity | $n$=10 | $n$=10 |
| first magnetism | $n$=10 | $n$=10 |
| weak student | | |
| first electricity | $n$=10 | $n$=10 |
| first magnetism | $n$=10 | $n$=10 |

The distribution of good and weak students over the cells was controlled for by asking the students, prior to assigning them to a condition, whether they had passed their first electricity & magnetism test or not. Since only about half of the sample had passed the test, this criterion led to equal sized successful and weak groups. (Notice that this -rough- criterion was used just for controlling the distribution of subjects over experimental conditions. In the analyses we used the criterion that was described in the previous section, and that takes into account performance on several more previous tests.)

### 2.2.2 The instruction for the experimental task

From previous research it appeared that the instruction that is given to the subjects may strongly influence the outcome of the experiment (Taconis, 1995). In the present experiment, the goal was to investigate how well the subjects were able to form meaningful categories based on solution methods and how well they were able to see to what category each problem belongs, instead of testing whether the subjects would perceive this type of ordering as the most natural one. Therefore, the instruction was to be quite explicit about the ordering principle that was intended in the experiment. A straightforward way of laying out the intended structure would be to give an example. Still, an example from an adjacent physics domain (say mechanics), was felt inappropriate because it would give too much information. For these considerations it was chosen to use as an instruction an example from an entirely different domain: cooking. The instructional text explained how a cook might answer when he would be asked what dishes where similar. Examples mentioned were cream of chicken soup and cheese sauce that were categorised together because both recipes involved preparing a roux, and an apple turnover and a savoury pie that both involve making pastry and baking in the oven. The instruction went on saying that the student was supposed to read all problem cards in the present set, prior to doing any sorting. When the sorting was

done the student would copy the numbers of all problems .n a category on a results form, together with the most appropriate name for that category. Some students asked what number of clusters they were supposed to distinguish; they were told that one big cluster containing all problems, or 20 separate clusters containing one problem each were considered undesirable.

Before starting, the students were told that the first set would take approximately 50 minutes to complete. Then there would be a coffee break of about 15 minutes after which the second set would be sorted in about 40 minutes. Subjects who couldn't finish the first task within 50 minutes were permitted to work on during the coffee-break. On the second task, subjects could work on until they were finished, but only a few people spent more than 40 minutes.

## 3. Results

### *3.1 Cluster analysis and analysis of pile labels, searching for a weak-beginner criterion*

The first question we will try to answer is whether the better and weaker students apply different sorting criteria from the criteria applied by the experts, or whether they try to apply the same criteria and are just less able to do so. In order to answer this question, we looked both at the clusters that emerged, and at the type of pile names that are given by the students. We performed this analysis both for the electrostatics problem set and with the magnetism problem set.

*Table 4 Average numbers of piles for all groups.*

| | | Electricity | Magnetism |
|---|---|---|---|
| experimenters' proposed order | *M* | 4 | 4 |
| other experts* | *M (SD)* | 6.67 (0.58) | 6.33 (2.08) |
| | *n* | 3 | 3 |
| good students | *M (SD)* | 5.97 (1.50) | 5.64 (1.51) |
| | *n* | 36 | 36 |
| weak students | *M (SD)* | 5.94 (1.73) | 5.69 (1.49) |
| | *n* | 35 | 35 |

*) The problem sets sorted by the experts were slightly larger than, and slightly different from, those sorted by the students

In order to compare the sortin,s for proficient students and weak students, the student level scale had to be dichotom.sed. We choose to draw the cut-off line at the median in order to get two equal sized groups. Out of the 80 subjects who participated in the problem sorting experiment, 9 subjects had either omitted a card from one of their problem sorting forms, had mentioned a card twice, or could not be classified as good or weak because of missing information. Therefore number of valid observations for all the analyses in this study will be $N = 71$. After applying the median split procedure we were left with a group of 36 proficient students and a group of 35 weak students. As a first descriptive we give the mean numbers of clusters we found for good and weak novices and for the experts for both problem sets in Table 4.

We used a hierarchical cluster analysis to study the clusters of problems that emerged in the proficient and weak students sortings respectively. For the purpose of this analysis we made no distinction between the elaborated and non-elaborated versions because that would leave use with too small groups to permit any sensible analysis. Moreover, we

have no reason to expect that the elaborated and non-elaborated versions would be sorted in fundamentally different ways.

The hierarchical cluster analysis procedure demanded that the data were structured in a particular way: for all the subjects in the experiment a correspondence matrix was computed for each problem sorting. Such a correspondence matrix consists of 20 rows, one for each problem card. A row consists of 20 positions, one for each problem card. A position gets the value '1' if both problems are placed in the same pile, or if the position is on the diagonal of the matrix. Otherwise the position gets a '0'. The correspondence matrices for all subjects were concatenated to a single file. From this file Euclidean distances between problems were computed for both the below and above median groups. Then a cluster analysis was performed using the 'average linking' algorithm.

The results were summarised in dendrograms that are displayed in appendix C. As can be seen from these dendrograms, the expert clusters can be clearly recognised in both the weak and the good students sortings. The main difference between good and weak student clusterings is that the distances within a cluster are larger for the weak student sortings, which implies that the problems within the cluster are less tightly bound together. In general were the student clustering deviates from the 'norm', there is at least one of the experts who 'deviated' in the same way.

*Table 5 Two frequently 'misplaced' problems*

| | |
|---|---|
| E14 Given are two parallel thin metal cylinders, carrying opposite charge Q and -Q. The length of both cylinders amounts 1 [m] and the distance between both cylinders amounts 1 [mm]. Compute the field in a position 1 meter away from the wires in the perpendicular bisectric plane to the cylinders (intended cluster: dipole approximation) | M11 Given is a densely wound flat coil that has N turns. The inner diameter of the coil is r the outer diameter of the coil is 2r. All turn of the coil are in the same plane. Compute the field at the centre of the coil (intended cluster: Biot-Savart's law) |

For the electricity problems, the good students' clustering except for one problem card (E14), was entirely according to the norm. The 'misplaced' card was also the card that had the weakest association to it's cluster. The resulting combinations with three problems in the cluster Coulomb's law/superposition were made by none of the experts. In the weak students' clustering three cards were placed in clusters different from the 'norm' (E5, E17, and E18). These three cards that were placed differently rendered only one combination that none of the experts made (E9 with E17). For the magnetism problems, the clusters for good and weak students were identical. Both the good and the weak students placed two cards (M7 and M11) in piles different from the 'norm'. Four of the resulting combinations of M11 and problems in the cluster 'Ampere' were made by none of the experts. Two of the cards that posed most problems to the novices are displayed in Table 5.

*Table 6 Taxonomy of category labels (items printed in italics do not represent the content labels themselves, but rather represent a group of content labels)*

| Type of label | Electricity label category | Magnetism label category |
|---|---|---|
| Strong physics procedure | Gauss' law<br>Gauss' law in differential form<br>Coulomb's law<br>Image charges<br>Dipole approximation | Ampere's law<br>Biot-Savart's law<br>Dipole approximation |
| General physics procedure | Superposition & *algebraic meth.*<br>Potential & *algebraic method*<br>Charge distribution & *method* | Superposition & *algebraic meth.* |
| Physical relation | Force in field<br>Superposition<br>Conductor & induced charge | Lorentz force<br>Superposition<br>Induction |
| Physics quantity & geometry | Field & *geometry*<br>Potential & *geometry* | Flux & *geometry*<br>Field & *geometry* |
| Physics quantity | Field<br>Potential<br>Charge density/distribution | Flux<br>Field |
| Geometry | Capacitor | *Geometry* |
| Algebraic procedure | Integration/*algebraic method* | Integration/*algebraic method* |
| Other content related labels | *Other content related label* | *Other content related label* |
| Not content related | *Not content related* | *Not content related* |
| 'I don't know' | I don't know | I don't know |

Though the cluster analysis did not suggest a fundamentally different 'weak beginner problem sorting criterion', we went on analysing the names subjects gave to the piles they made, searching for the 'weak beginner criterion'. Because this is a more laborious type of analysis, we did this analysis only for a subset of the data. We took the labels invented by 10 subjects who were randomly selected from the 20 most proficient subjects, and compared them to the labels that were given by 10 subjects who were selected from the 20 least proficient subjects. Since all the subjects had made two problem sortings, we had two collections of labels, one for the electricity problems and another for the magnetism problems. In total the selected subjects had named 107 clusters for the electricity problems, and 105 clusters for the magnetism problems. In order to make comparison between labels possible, first all labels with an equivalent meaning had to be clustered together. This was done by two physics experts independently, after which differences were resolved by discussion. During the process the experts did not know whether a label was given by a good or by a weak student. The experts agreed that there were 20 different meanings in the electricity labels, with 7 labels remaining unclassified. For the magnetism problems, there were 14 categories of meaning, with 13 labels remaining unclassified. The resulting content categories were then classified according to the type of information they expressed. The resulting scheme is displayed in Table 6.

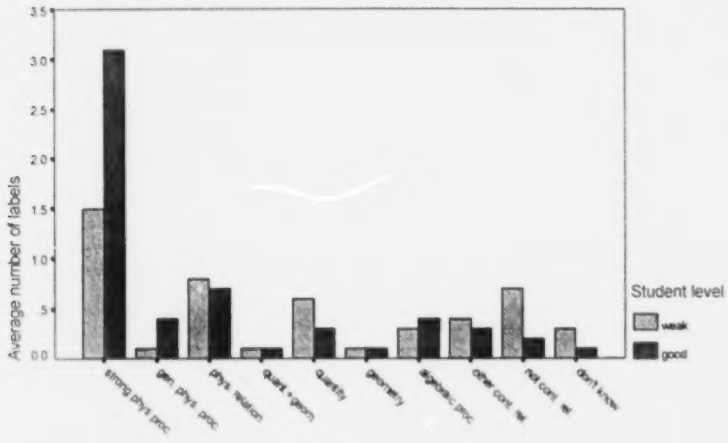



*Figure 2 Distribution of labels for electricity problems*

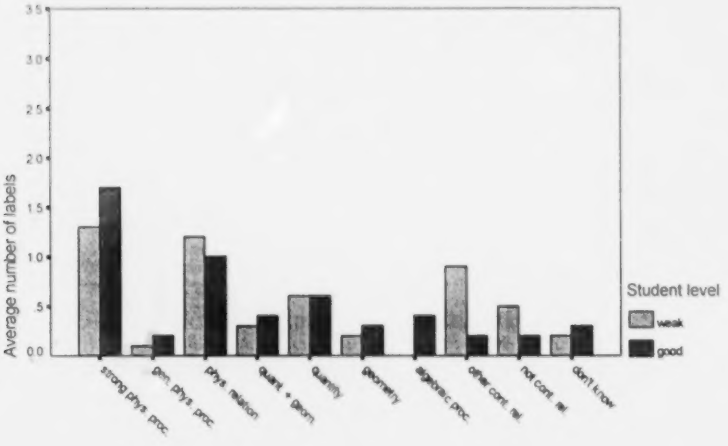



*Figure 3 Distribution of labels for magnetism problems*

The proportions of labels in the different categories are displayed in Figure 2 and Figure 3 for electricity and magnetism problems respectively. Apparently for both good and weak students the proportion of 'strong physics procedures' labels is highest. The main differences between good and weak students are that good students tend to mention more 'strong physics procedures' and the weak students tend to mention more 'not content related' labels. The latter effect is mainly accounted for by a single individual in the weak students group, who used not content related labels exclusively.

## *3.2 Analysis of piles, similarity to expert sorting*

### 3.2.1 The problem sorting performance-scale and descriptives

Now that we have demonstrated that the cluster solution for the good and weak beginner groups as a whole are similar to each other and to the experts solution, we want to assess the quality of individual sortings in order to test whether the elaborations had an effect. We therefore constructed a formula that expresses the similarity between sortings in a single numeral. From the analysis of the expert sortings, it appeared that some pairs of problems occur frequently in expert sortings and that some combinations never occur. A combination was judged to occur frequently among experts if the experimenters and at

least two out of the three other experts had made the combination. The similarity measure was based on these two groups of pairs; all other combinations were neglected in computing the expert likeness. Thus for each subject we had two scores per sorting:

$N_{exp\text{-}like}$: the number of combinations the subject made, that are made by -almost- all experts too,
$N_{not\ exp\text{-}like}$: the number of combinations the subject made, that no expert makes.

For each set of card we had two normalisation parameters:

$N_{exp\text{-}always}$: the number of combinations that are made by -almost- all experts,
$N_{exp\text{-}never}$: the number of combinations that no expert makes.

The resulting 'expert likeness' score 'E' for subject 'i' was computed from the following formula:

$$E_i = \frac{N_{i,\ exp\text{-}like}}{N_{exp\text{-}always}} - \frac{N_{i,\ not\ exp\text{-}like}}{N_{exp\text{-}never}}$$

The denominators in both fractions are different for the set of electricity problems and the set of magnetism problems (values can be found in appendix B). The numerators are measures of a particular students' problem sorting. The score according to this formula is a rather well behaved similarity measure; a 'perfect' sorting would yield the maximum score: '1', all problems thrown together in a single pile would give '0' as an outcome, which would also be the result if each problem is sorted in a separate, one card, pile.

A remarkable difference between the electricity and the magnetism problem sets is that, though in both sets a comparable -large- number of combinations is not expert like, the numbers of problems that are put together by all experts is rather different, being 32 for electricity and only 18 for magnetism problems. As a consequence the score for magnetism problems is less robust than the score for electricity problems is.

To test the expert likeness score E's sensitivity to random variations, and to compare the performance of the subjects the average score for a 'blind' sorting, we generated a set of 1000 random sortings. In these sortings the number of piles per sorting was distributed binominally with the average number of clusters set to 5.9, which is closely corresponding to the average number of piles in the real data (Table 4). Both the scores for real novices and the artificially generated scores are described in Table 7. From these data we conclude that, though the scores for magnetism problems indeed show a greater variance than the scores for the electricity problems do, for both the electricity and the magnetism cards the average random scores are clearly small compared to the real scores.

*Table 7 Means and standard deviations of the expert likeness score* E *per experimental group for real data and for artificially generated random data.*

| | | Electricity | | Magnetism | |
|---|---|---|---|---|---|
| | | without elaboration | with elaboration | without elaboration | with elaboration |
| weak students | *M (SD)* | .31 (.14) | .23 (.14) | .31 (.20) | .48 (.21) |
| | *n* | 16 | 19 | 19 | 16 |
| good students | *M (SD)* | .36 (.22) | .58 (.21) | .46 (.25) | .57 (.25) |
| | *n* | 19 | 17 | 17 | 19 |
| random data | *M (SD)* | .0012 (.08) | | .0019 (.10) | |
| | *N* | 1000 | | 1000 | |

### 3.2.2 Effect of student level and elaboration

Problem sorting performance was supposed to be dependent on the level of the student, on whether or not the problem was presented in an elaborated format, and on the interaction between both factors. In order to explore the effect of elaborations, we need to compare scores on electricity sortings to scores for magnetism problems. Therefore, the scores on all electricity problems were converted to z-scores and the same was done for magnetism problems. As a consequence the systematic difference in scores between magnetism and electricity problem sortings was eliminated. The data are summarised in the following graph:

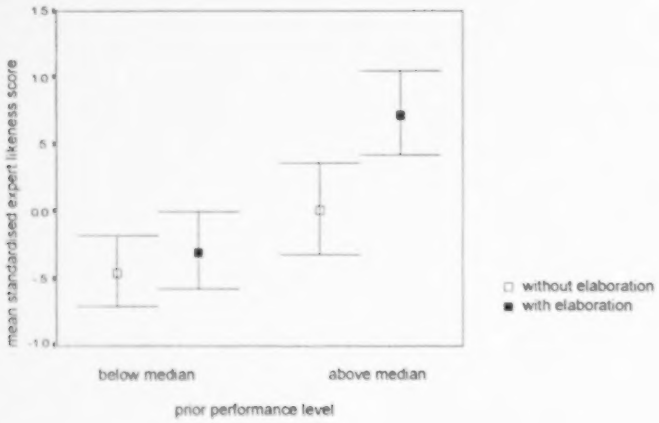



*Figure 4 Expert likeness score by student level. The values along the vertical axis are standardised scores. The vertical bars indicate the 95% confidence interval for the mean.*

As can be seen from the graph, the performance of a student on the sorting task increases with increasing student level for non-elaborated as well as for elaborated problem formats.

In the experiment we had two blocking factors namely the experimental task version and the order of the tasks. In a within subjects design, the main effects of these factors are accounted for implicitly by the factor 'subject'. Interactions between the blocking factors and the experimental factors were not expected. Therefore they were not included in the analysis.

A repeated measures analysis confirmed that there was a main effect for student level ($F(1, 69) = 19.0$, $p < .001$) and also a main effect for the presence of elaborations in the material ($F(1, 69) = 12.0$, $p = .001$). Moreover, the interaction between student level and the presence of elaboration could be confirmed ($F(1, 69) = 5.1$, $p = .028$). A further analysis within the proficient and less-proficient subgroups shows that for proficient students there is an effect of the presence of elaborations ($F(1, 69) = 16.5$, $p < .001$), whereas for the less proficient students we could demonstrate no effect at all ($F(1, 69) = 0.72$, $p = .398$).

The effect sizes, in terms of explained variance, were moderate: main effect of student level: $\eta^2 = .216$, main effect of the presence of elaborations: $\eta^2 = .148$, interaction between student level and the presence of elaborations: $\eta^2 = .068$. We consider these effect-sizes to be large enough to be relevant to psychological theory.

## 4. Conclusion & discussion

Our hypothesis was that all our subjects would have some knowledge of problem types and their solutions, and that their major problem was to choose the proper problem type for a given situation. In line with this hypothesis we found that the clustering solution for the beginners groups basically resembles the expert clustering, and that beginners often give the clusters proper names also. We found no evidence of a systematically deviant 'beginner sorting', but rather random failures to see the proper problem type. Thus the conclusion by Chi et al. (1981) that novices sort according to superficial criteria and name their categories accordingly, should be qualified to apply in particular conditions only. As noted earlier by Taconis (1995) two factors may explain for the different results: the instructional format and the composition of the problem set.

We had predicted that the availability of simple elaborations would be more helpful to weak beginners than they would be to good beginners, but we found the opposite. As we have pointed out, the elaborations that we provided would be helpful in the process of analysing, restructuring and solving the problem. Our tentative conclusion is that the weak students had no model of the situation at hand that could serve to integrate the given elaborations into a coherent whole. A text base or discourse representation would be of little help in doing so, whereas a situation model would. This is because the given elaborations were redundant from a physics point of view, but not from a discourse point of view, as they appealed to external physics knowledge. In the present experiment it was taken for granted that all subjects mastered the knowledge underlying the elaborations. This is a reasonable assumption since the elaborations presented were very simple. To rule out the possibility that the weak students simply missed the conceptual physics knowledge that would be necessary to comprehend the elaborations presented, in a further experiment this assumption could be subject to direct measurement.

There are two possible mechanisms that would explain how proficient students do profit from the elaborations we gave: either they had already inferred a corresponding elaboration themselves, or they had not. In the first case, the supportive effect of giving elaborations should be explained in terms of focusing attention to, or increasing activation of a particular piece of information. In the latter case, the elaboration really added something to the problem representation. The two mechanisms could be distinguished experimentally by first presenting a 'minimum' problem descriptions, and then presenting an elaboration. The time a subject uses to interpret an elaboration he already inferred would be shorter than the time required to interpret an elaboration that was not inferred yet (Noordman, 1992).

Because the type of elaborations that we gave could be useful only to someone who is trying to induce a situation model, we interpret the current results to imply that good students are inducing a situation model from their text base representation, whereas weak students are unable to induce a situation model. This conclusion is in line with results by De Jong and Ferguson-Hessler (1991) who had their subjects reconstruct problem statements that were presented only briefly. They found no difference between good and weak students when it came to reconstruction of the information in the same format that it was presented in, whereas if the reconstruction was to be in a different mode than the mode it was presented in (verbal vs. visual), results of good students proved to be better. Since the latter task would require a mental model of the situation, whereas the first task could be done relying on a text base representation, they concluded that weak students did not construct a mental model of the situation.

From the present findings, we conjecture that weak students, being unable to induce a situation model from text, may be better off if, instead of having to induce a situation model from scratch, they are provided with a representation that matches the inherent

structure of the situation model much closer, namely a situation representation in the figural mode. If a weak student would have a proper type of drawing available in addition to the textual description of the problem, he could infer relations between the two deductively, which would be less demanding to his constructive abilities. This hypothesis could be tested in a further experiment that basically would be a replication of the current experiment, apart from providing diagrams of the problem as an elaboration, instead of the textual elaborations that we employed in the current experiment.

This study suggests that, for weak students, the pay-off of a more extensive analysis of the problem statement would be small. This finding sheds new light on the disappointing results and especially the lack of transfer of general-problem-solving-skills programs (Mansfield, Busse, & Krepelka, 1978; Feuerstein, 1980; Mettes & Pilot, 1980; De Jong, 1986). The present study suggests that well-chosen content-related support may be more successful in advancing the reasoning process during problem solving.

To conclude, we propose that verbal reasoning deserves attention as an important phase in the construction of a situation model in -physics- problem solving.

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# Appendix A. Examples of problem statements plus elaborations

## *Problem 1: The field of two concentric cylinders*

**A minimal problem statement, containing only a minimum of 'deep' information:**
Given two copper cylinders, $o_1$ and $o_2$, with radiuses $r_1$ and $r_2$, where the relation between the radiuses is given by $r_2=3r_1$. Both cylinders are aligned along the x-axis. $O_2$ is grounded; $o_1$ carries a charge $q_1$ per meter. Compute the potential at the surface of $o_1$.

**Formal description**

give{situation$_1$,

| | | |
|---|---|---|
| shape($o_1$)=cylinder | (1) | |
| shape($o_2$)=cylinder | (2) | |
| shape.radius($o_1$)=$R_1$ | (3) | |
| shape.radius($o_2$)=$R_2$ | (4) | |
| $3.R_1=R_2$ | (5) | |
| medium($o_1$)=copper | (6) | |
| medium($o_2$)=copper | (7) | |
| x_axis(h1) | (8) | *(By the introduction of an axis, it is avoided that the relation between the two cylinders should be given directly.)* |
| orientation($h_1$, $o_1$)=concentric | (9) | |
| orientation($h_1$, $o_2$)=concentric | (10) | |
| connect($o_2$, ground) | (11) | |
| charge.per_meter($o_1$)=$Q_1$} | (12) | |

goal{ situation$_1$,

| | | |
|---|---|---|
| electrostatic_potential($o_2$)} | (13) | |

**Extra information that can be added**

| *From:* | *It follows that:* | | *By the use of inference:* |
|---|---|---|---|
| 6 ⇒ | medium($o_1$)=conductor | (14) | *copper is a conductor* |
| 7 ⇒ | medium($o_2$)=conductor | (15) | *copper is a conductor* |
| 12 ⇒ | e_field($o_3$) | (16) | *a charge induces an electric field* |
| 9∧10 ⇒ | orientation($o_1$, $o_2$)=concentric | (17) | *concentrism is a transitive property* |
| 5∧9∧10 ⇒ | position($o_1$, $o_2$)=in | (18) | *'concentric with' and 'smaller than' implies 'in'* |
| 1∧2∧9∧10⇒ | symmetry(situation$_1$)=cylindrical | (19) | *if all elements are cylindrical and placed concentric to each other, the entire situation is cylinder symmetric* |
| 5∧7∧9∧10∧11⇒ | region($h_1$) | | |
| | position($h_1$,$o_2$)=outside | | |
| | atposition($h_1$,e_field.magnitude())=0 | (20) | *a grounded conductor shields its inside from external influences and vice versa* |
| 5∧7∧9∧10∧11 ⇒ | | | |
| atposition($h_1$,electrostatic_potential($o_3$))=0 | | (21) | $V=\int Eds$ |
| 7∧11 ⇒ | region($h_2$) | | |
| | position($h_2$,$o_2$)=outer_surface | | |
| | atposition($h_2$,charge.per_meter($o_2$))=0 | (22) | ? |
| 5∧7∧9∧10∧12 ⇒ | region($h_3$) | | |
| | position($h_3$,$o_2$)=inner_surface | | |
| | atposition($h_3$,charge.per_meter($o_2$))=-$Q_1$ | (23) | *the net charge enclosed in a closed surface that lies completely within a conductor amounts to zero* |
| 5∧7∧9∧10∧11∧12 ⇒ | charge($o_2$)= -charge($o_1$) | (24) | *the total charge on a grounded system amounts to zero* |
| 1∧5∧9∧10∧12 | | | |

⇒ region($h_4$)
position($h_4$, $o_1$)=outside
position($h_4$, $o_2$)=inside
atposition($h_4$, e_field($o_3$))≅$Q_1/r$ (25) *inside a (hollow) conductor, the charge distribution outside the conductor has no influence, thus the field is the field of the inner cylinder*

1∧5∧9∧10∧12
⇒atposition($h_4$, elec_potential($o_3$)) ≅$-Q_1$ ln r (26) *see 25*

5∧7∧9∧10∧12
⇒atposition($h_1$→ $h_3$, potential($o_3$))= atposition($h_4$→ $h_3$, potential($o_3$))
(27) *the potential runs continuously across a surface charge*

There are major differences in depth between the various statements. Statement 14 for example follows from the simple rule copper is a conductor, combined with the information that object$_1$ is made of copper. Statement 23 in contrast is based on the following: *inside a conductor the field amounts the zero,* plus *the total flux through a surface is proportional to the total charge enclosed,* plus some knowledge about geometry rules, plus information from statements 5, 7, 9, 10 and 12 and the generalisations used in inferring 15, 17 en 19.
Upon derivation off the final statement, the answer is within reach, all that is to be done is to fill in formulas and carry out computations.

## *Problem 2: A conducting sphere in an external field*

**A minimal problem statement, containing only a minimum of 'deep' information:** A conducting sphere is placed in an initially homogeneous electric field (parallel to the x axis). Compute the electric field.

**Formal description**

give{situation$_1$,
e_field($o_1$)=homogeneous.vector (1)
x_axis(h1) (2)
orientation($h_1$, $o_1$)=parallel} (3)
give{situation$_2$,
situation$_1$ interacts revise initial field description!
shape($o_2$)=sphere (4)
shape.radius($o_2$)=$R_1$ (5)
medium($o_2$)=conductor (6)
charge.magnitude($o_2$)=0} (7)
goal{situation$_2$, shape(o1)} (8)

From a discourse point of view, this representation might be acceptable, from a physics point of view it hardly is because (1) the description suggests a changing situation and (2) the final situation is not clearly defined. A better description of the final state would make use of the boundary conditions that are implicit in the initial description. Thus the inference of boundary conditions is one of the elaborations that is to be made to come from the initial problem description to an acceptable physics model:

**Extra information that can be added**

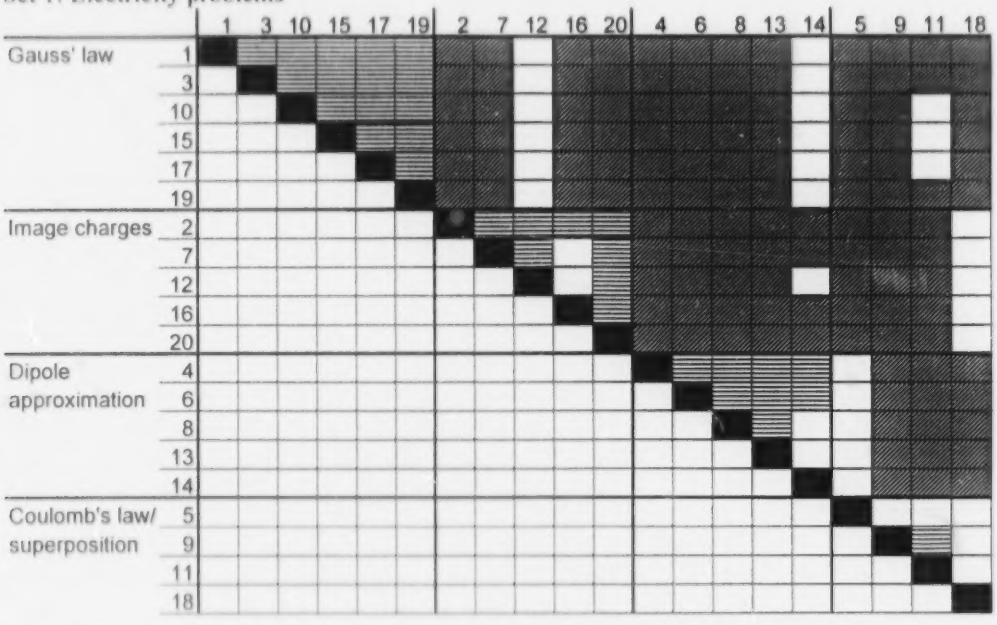
| *From:* | | *It follows that:* | | *By the use of inference:* |
|---|---|---|---|---|
| 4∧6 | ⇒ | region($h_1$)<br>position($h_1$,$o_2$)=inside<br>e_field(situation$_2$)= e_field($o_1$)+e_field($o_2$)<br>atposition($h_1$,e_field.magnitude(situation$_2$))=0 | (9) | *the field inside a conductor amounts to zero* |
| 1∧9 | ⇒ | field.magnitude($o_2$)≠0 | (10) | |
| 10 | ⇒ | atposition($h_3$,charge.per_cubic($o_2$))≠0 | (11) | *a field is caused by a charge distribution* |
| 1∧4 | ⇒ | region($h_2$)<br>position($h_2$,origin)=(x=∞)<br>atposition($h_2$,e_field(situation$_2$))=homogeneous.vector | (12) | *a finite charge distribution gives only a local effect* |
| 1∧4 | ⇒ | region($h_3$)<br>position($h_3$,origin)=(x=-∞)<br>atposition($h_3$,e_field(situation$_2$))=homogeneous.vector | (13) | *see 12* |
| 6 | ⇒ | region($h_4$)<br>position($h_4$,$o_2$)=outer_surface<br>atposition($h_4$,perpendicular($h_4$, field(situation$_2$))) | (14) | *the electric field a the surface of a conductor is perpendicular to the conductor surface* |
| 14∧ | ⇒ | region($h_5$)<br>position($h_5$,origin)=(x=0)<br>atposition($h_5$∩$h_4$,e_field(situation$_2$))=0 | (15) | *symmetry considerations* |

Now that the problem has been transformed into a proper boundary value problem, it can be solved either by changing to spherical coordinates and fitting a multipole series expansion to the boundary conditions, or by seeing that the initial field description can be replaced with two charged parallel infinite flat plates in x=∞ and x=-∞ respectively. Then the relation between field and surface charge would the be: $E = \sigma / \varepsilon_0$. This would result in an image dipole in the sphere.

By applying (15) we could then find the magnitude of the dipole: $\vec{p} = 8\pi\varepsilon_0 E_0 R^3$, thus the resulting field would be: $\vec{E} = \vec{E}_0 + \frac{1}{8\pi\varepsilon_0}\frac{\vec{p}}{\vec{r}^3}$. The latter solutions route involves major restructuring of the problem representation again.

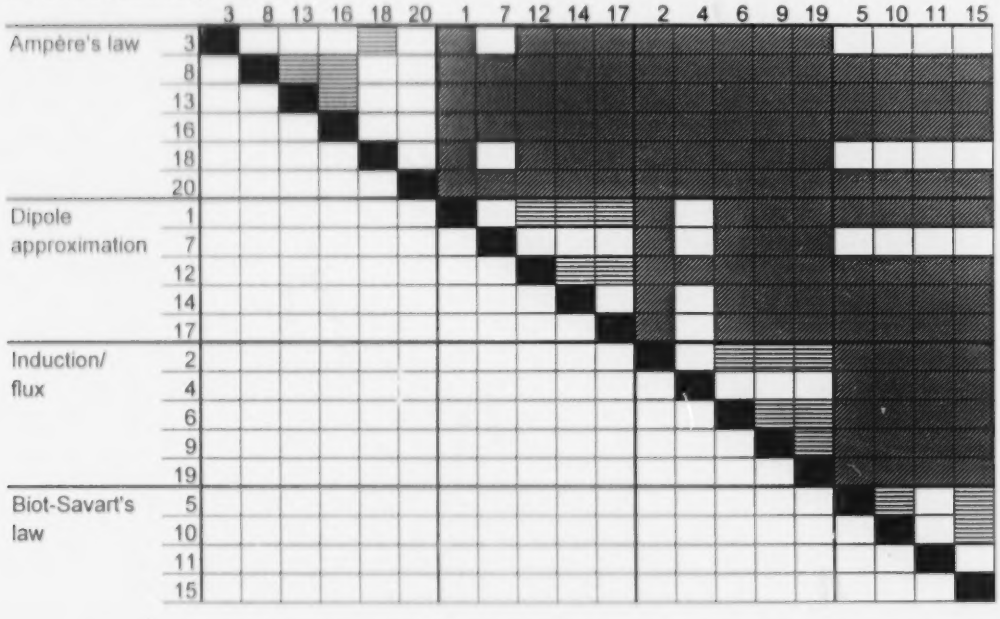
# Appendix B. Matrices for experts sortings

Set 1: Electricity problems




*(Maximum number of expert-like combinations: 32. Maximum number of not expert-like combinations: 123)*

Set 2: Magnetism problems




*(Maximum number of expert-like combinations: 18. Maximum number of not expert-like combinations: 131)*

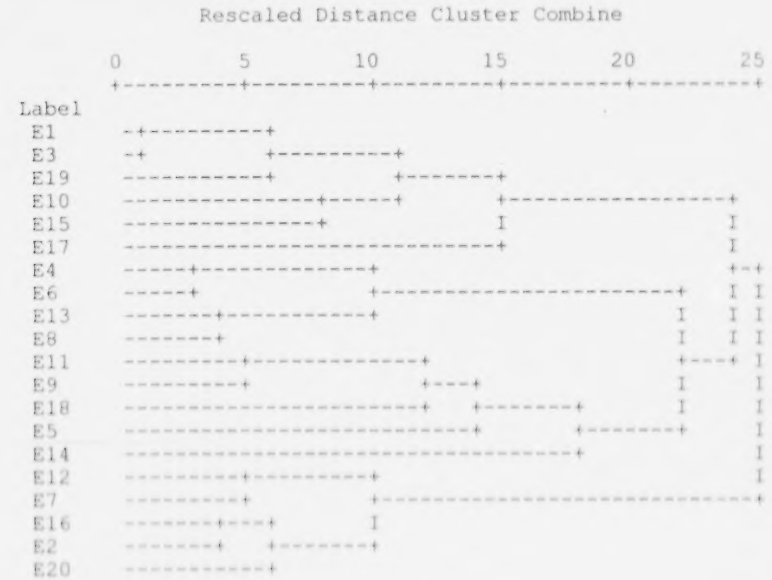
Legend

the experimenters and at least 2 out of 3 other experts placed these problems in the same pile

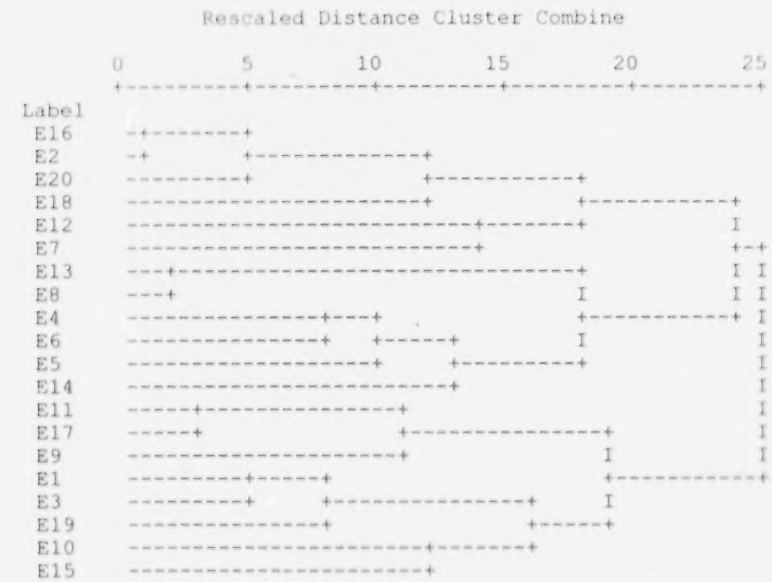
neither the experimenters nor one of the other experts placed these problems in the same pile

# Appendix C. Dendrograms

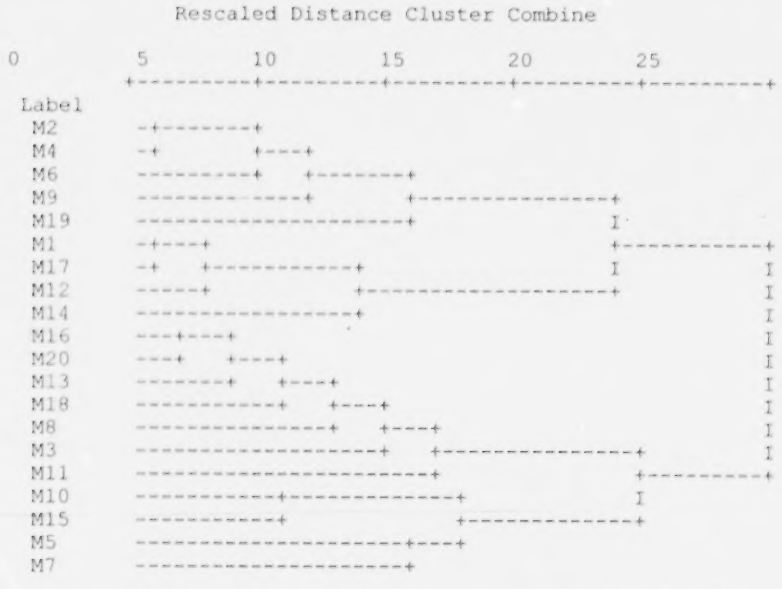
In the following dendrograms the label numbers correspond to the label numbers in appendix B. The distance measure used is Euclidean distances. As a clustering method, average linking between groups was applied.



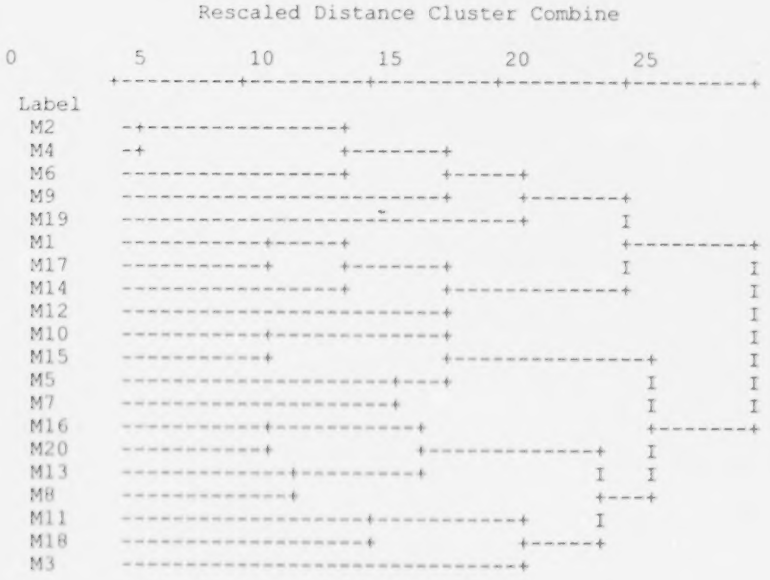

*Dendrogram of good students sortings of electricity problems (based on 36 subjects).*




*Dendrogram of weak students sortings of electricity problems (based on 35 subjects).*




*Dendrogram of good students sortings of magnetism problems (based on 36 subjects)*




*Dendrogram of weak students sortings of magnetism problems (based on 35 subjects)*